C PROGRAM FOR RUTHERFORD SCATTERING OF ALPHA PARTICLE

_A PROJECT REPORT SUBMITED TO

SAVITRIBAI PHULE PUNE UNIVERSITY, PUNE

YEAR 2020-21

FOR THE DEGREE OF

MASTER OF SCIENCE IN PHYSICS

BY

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UNDER THE GUIDANCE OF

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CERTIFICATE

This is to certify that the project report wok entitled

"C PROGRAM FOR RUTHERFORD SCATTERING OF ALPHA PARTICLE"

Has been carried out successfully by

MR.MAGAR PRATIK KISHOR

During the year 2020-21 as a M.Sc project

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ACKNOWLEDGEMENT

It gives me great pleasure on bringing my project entitled " **c program for Rutherfords scattering of alpha particle**". I greatly accept this opportunity to convey my heartiest thanks and express my deep sense of gratitude to my project guide

I must acknowledge the financial support given to this project by my parents without which it would have been difficult to complete the work in time.

I would like to thank all my classmates whose help and criticism made me able to better my project report. My ideas were shaped and refined progressively through my discussions with them from time to time.

Pratik m.

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M.SC ll

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INTRODUCTION TO RUTHERFORD SCATTERING:

Ernest Rutherford (1871-1937)

Rutherford scattering phenomenon is proposed by Ernest Rutherford in 1911 by performing his famous experiment of scattering of α-particles by a gold foil about 1000 atoms thick. This revolutionary phenomenon changed our understanding of an atom and led to the development of Rutherford model and Bohr model of an atom. In this experiment, fast moving α-particles were made to fall on a thin gold foil. The reason of selecting gold foil is to have as thin as layer is possible. α-particles ($_2He^4$ or $_2\alpha^4$) are doubly-charged helium ions. Since they have mass of 4u, the fast moving alpha particles have a considerable amount of energy. Since the alpha particles are much heavier than protons, it is not expected to see large deflections. But, the α-particle scattering experiment gave totally unexpected results. It is observed that most of the fast moving α-particles passed straight through the gold foil. Some of α-particles were deflected by the foil by small angles and a very small angles and a very few appeared to be back scattered.

Rutherford scattering is the elastic scattering of charged particles by the Coulomb interaction. It is a physical phenomenon that led to the development of the planetary Rutherford model of the atom and eventually the Bohr model. Rutherford scattering was first referred to as Coulomb scattering because it relies only upon the static electric (Coulomb) potential, and the minimum distance between particles is set entirely by this potential. The classical Rutherford scattering process of alpha particles against gold nuclei is an example of "elastic scattering" because neither the alpha particles nor the gold nuclei are internally excited. The Rutherford formula further neglects the recoil kinetic energy of the massive target nucleus.

Rutherford's Alpha Scattering Experiment:

Rutherford conducted an experiment by bombarding a thin sheet of gold with α-particles and then studied the trajectory of these particles after their interaction with the gold foil.

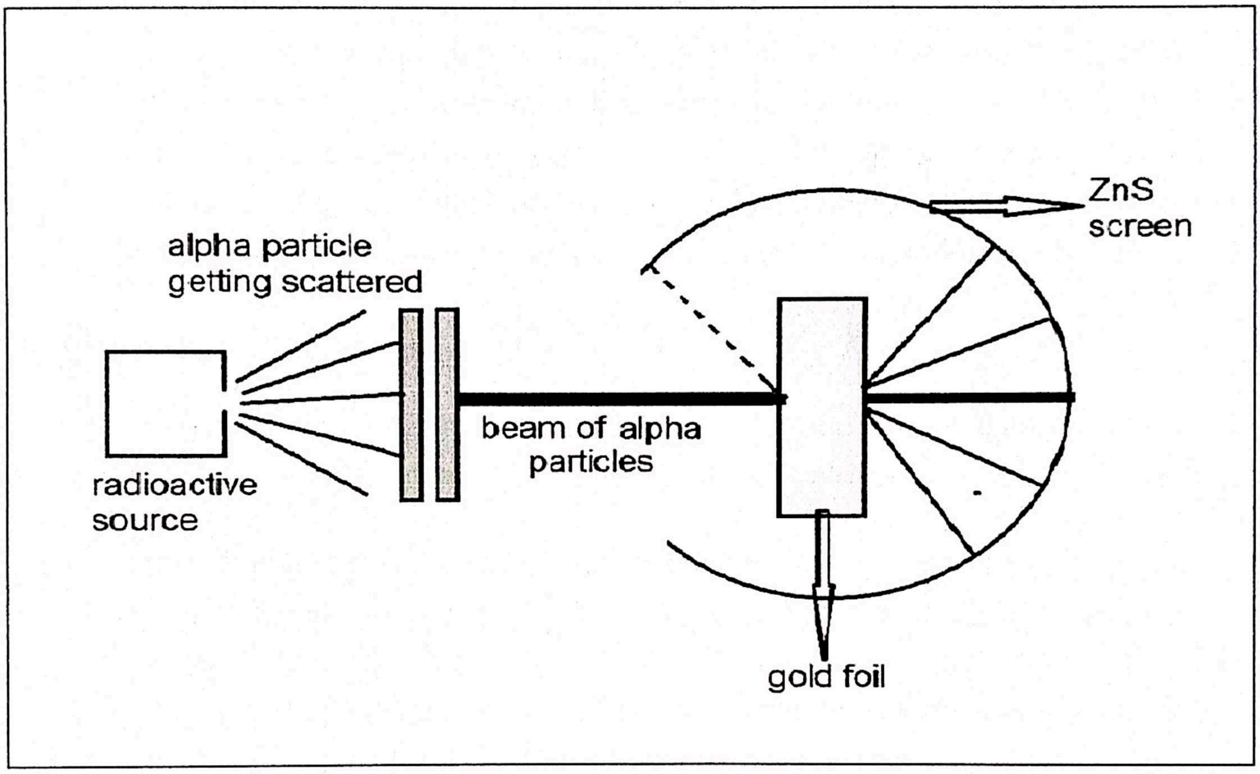


Fig. Experimental arrangement of Rutherford Scattering expt.

Rutherford, in his experiment, directed high energy streams of α-particles from a radioactive source at a thin sheet (100 nm thickness) of gold. In order to study the deflection caused to the α-particles, he placed a fluorescent zinc sulphide screen around the thin gold foil. Rutherford made certain observations that contradicted Thomson' s atomic model.

Observations of Rutherford's Alpha Scattering Experiment:-

The observations made by Rutherford led him to conclude that:

1. A major fraction of the α-particles bombarded towards the gold sheet passed through it without any deflection, and hence **most of the space in an atom is empty**.
2. Some of the α-particles were deflected by the gold sheet by very small angles, and hence the **positive charge** in an atom **is not uniformly distributed. The positive charge** in an atom **is concentrated in a very small volume.**
3. Very few of the α-particles were deflected back, that is only a few α-particles had nearly 180o angle of deflection. So the **volume occupied by the positively charged particles in an atom is very small as compared to the total volume of an atom.**

Rutherford's Atomic Model

Based on the above observations and conclusions, Rutherford proposed the atomic structure of elements. According to the Rutherford atomic model:

1. The positively charged particles and most of the mass of an atom were concentrated in an extremely small volume. He called this region of the atom as a nucleus.
2. Rutherford model proposed that the negatively charged electrons surround the nucleus of an atom. He also claimed that the electrons surrounding the nucleus revolve around it with very high speed in circular paths. He named these circular paths as orbits.
3. Electrons being negatively charged and nucleus being a densely concentrated mass of positively charged particles are held together by a strong electrostatic force of attraction.

Runge-kutta method for Fourth order:

In numerical analysis, the Runge–Kutta methods are a family of implicit and explicit iterative methods, which include the well-known routine called the Euler Method, used in temporal discretization for the approximate solutions of ordinary differential equations. These methods were developed around 1900 by the German mathematicians Carl Runge and Wilhelm Kutta.

The most widely known member of the Runge–Kutta family is generally referred to as "RK4", the "classic Runge–Kutta method" or simply as "the Runge–Kutta method".

Consider the differential equation,

$$\frac{dy}{dx} = f(x,y), y(x_0) = y_0$$

Then fourth order Runge-Kutta method yields,

$$y(x+h) = y(x) + \frac{1}{6}[k_1+2k_2+2k_3+k_4]$$

Where;

$$k_1 = hf(x,y)$$

$$k_2 = hf\left(x + \frac{h}{2}, y + \frac{k_1}{2}\right)$$

$$k_3 = hf\left(x + \frac{h}{2}, y + \frac{k_2}{2}\right)$$

$$k_4 = hf(x+h, y+k_3)$$

- k_1 is the slope at the beginning of the interval, using y ;
- k_2 is the slope at the midpoint of the interval, using y and k_1;
- k_3 is again the slope at the midpoint, but now using y and k_2 ;
- k_4 is the slope at the end of the interval, using y and k_3

Mathematical background for Rutherford scattering:

Consider the scattering of a single α-particle by the single gold nucleus due to Coulomb force only. Let the moving α-particle be a point charge of a charge $q\alpha$ and the gold nucleus also be a point chargeq_{Au}. The gold nucleus is called target and α-particle is called a projectile. The force of interaction between an α-particle and the gold nucleus is the Coulomb force given by

$$\vec{F}(t)=\frac{q_\alpha q_{Au}}{4\pi\varepsilon_0}\frac{\vec{r_\alpha}(t)-\vec{r}_{Au}}{|\vec{r_\alpha}(t)-\vec{r}_{Au}|^3} \qquad (1)$$

Where $\vec{r}_\alpha$ and $\vec{r}_{Au}$ are the position vectors of α-particle and gold nucleus, respectively, ε_0=8.855*$10^{-12}\frac{C^2s^2}{m^2kg}$ is the permittivity of free space. Let $q_\alpha = Z_\alpha e$ and $q_{Au} = Z_{Au}e$, where Z_α and Z_{Au} are atomic numbers of α-particle and gold nucleus, respectively, and e=1.602×10^{-19}C

Is the charge of a proton. Using Newton' s second law, F=ma=$m_\alpha\frac{d^2\vec{r}_\alpha}{dt^2}$, the acceleration, $a = \frac{d^2\vec{r}_\alpha}{dt^2}$, of the α-particle is given by

$$\frac{d^2\vec{r}_\alpha}{dt^2} = \frac{Z_\alpha Z_{Au}e^2}{4\pi\varepsilon_0 m_\alpha}\frac{\vec{r}_\alpha(t)-\vec{r}_{Au}}{|\vec{r}_\alpha(t)-\vec{r}_{Au}|^3} \qquad (2)$$

Where $m_\alpha = 6.645 * 10^{-27}$kg is the mass of α – particle.

This second order differential equation describes a trajectory of motion of the α-particle due to the presence of a gold nucleus. For simplification and a better visualization of the problem, let us consider the problem in two dimensions. Hence in a component form equation (2) can be written as

$$\frac{d^2(x_\alpha\hat{\imath}+y_\alpha\hat{\jmath})}{dt^2} = \frac{Z_\alpha Z_{Au}e^2}{4\pi\varepsilon_0 m_\alpha}\frac{(x_\alpha-x_{Au})\hat{\imath}+(y_\alpha-y_{Au})\hat{\jmath}}{|(x_\alpha-x_{Au})\hat{\imath}+(y_\alpha-y_{Au})\hat{\jmath})|^3} \qquad (3)$$

Let $(x_\alpha - x_{Au}) = x$ and $(y_\alpha - y_{Au}) = y$. Hence, the above equation becomes

$$\frac{d^2(x_\alpha\hat{\imath}+y_\alpha\hat{\jmath})}{dt^2} = \frac{Z_\alpha Z_{Au}e^2}{4\pi\varepsilon_0 m_\alpha}\frac{x\hat{\imath}+y\hat{\jmath}}{|x\hat{\imath}+y\hat{\jmath}|^3} = \frac{Z_\alpha Z_{Au}e^2}{4\pi\varepsilon_0 m_\alpha}\frac{x\hat{\imath}+y\hat{\jmath}}{(\sqrt{x^2+y^2})^3} \qquad (4)$$

In the above equation, magnitudes of various quantities change over several orders. Hence, to avoid computational errors, let us express the above equation in dimensionless units. For this, let us express time in units of 10^{-22}sec i.e. 1 unit time of simulation corresponds to 10^{-22}sec.

Hence, $t_{exp} = t_{simu} * 10^{-15}$m. Hence, the conversion relation for the velocity will be

$$v_{exp}\frac{m}{sec} = v_{sim}\frac{10^{-15}}{10^{-22}} = v_{sim} * 10^7.$$

Let $B_{exp} = \frac{e^2}{4\pi\varepsilon_0 m_\alpha} = \frac{(1.602*10^{-19})^2 C^2}{4*3.141593*8.854*10^{-12}*6.645*10^{-27}} = 0.03471208\frac{m^3}{sec^2}$

Hence, $B_{sim} = B_{exp} * \frac{10^{-44}}{10^{-45}} = 0.3471208$. Hence equation (4) in dimensionless form becomes

$$\frac{d^2(x_\alpha\hat{\imath}+y_\alpha\hat{\jmath})}{dt^2} = B_{sim}Z_\alpha Z_{Au}\frac{x\hat{\imath}+y\hat{\jmath}}{(\sqrt{x^2+y^2})^3} \qquad (5)$$

The above second order equation can be expressed in terms of the following coupled first ordered equations

$$\frac{dvx_\alpha}{dt} = B_{sim}Z_\alpha Z_{Au}\frac{x}{(\sqrt{x^2+y^2})^3}$$

$$\frac{dx_\alpha}{dt} = Vx_\alpha$$

$$\frac{dvy_\alpha}{dt} = B_{sim}Z_\alpha Z_{Au}\frac{y}{(\sqrt{x^2+y^2})^3}$$

$$\frac{dy_\alpha}{dt} = Vy_\alpha$$

We will solve these equations using the fourth order Runge-Kutta method. In program we place the gold nucleus at (0, 0) position and the α-particle moves from left to right along a positive x-direction towards the gold nucleus.

Algorithm:

1. Start
2. Define functions $f(x) = \frac{xB_{sin}z_\alpha z_{Au}}{(\sqrt{x^2+y^2})^3}$, $f(x_1) = Vx_\alpha$, $f(y) = \frac{xB_{sin}z_\alpha z_{Au}}{(\sqrt{x^2+y^2})^3}$, and $f(y_1) = V_{y\alpha}$.
3. Place the target and projectile at positions (x_t, y_t) and (x_p, y_p) respectively.
4. Define final time t_{max}, time step h, mass of an α-particle m_α, and y-component of initial velocity of the projectile as V_{y0}=0.
5. Read energy of α-particle E, in ev and convert it in joules.
6. Find x-component of velocity of the projectile using $V_{x0}=\sqrt{2Em_\alpha}$
7. Scale the velocity using relation $V_{x0} = V_{x0} * 10^{-7}$.
8. Find the relative vector (x_0, y_0) using relations $x_0 = x_p - x_t$ and $y_0 = y_p - y_t$
9. Calculate initial distance of α-particle from target using formula dist=$\sqrt{x_0{}^2 + y_0{}^2}$ $*10^{-15}$
10. Print t_0, x_p *and* y_p
11. **For** $t = t_0 + h$ *to* t_{max} in steps of h **do**
12. $k_1 = h * fx(t_0, x_0, y_0, z_p, z_t)$
13. $k_2 = h * fx(t_0 + 0.5 * h, x_0 + 0.5 * k_1, y_0 + 0.5 * p_1, z_p, z_t)$
14. $p_2 = h * fx1(v_{x0} + 0.5 * k_1)$
15. $k_3 = h * fx(t_0 + 0.5 * h, x_0 + 0.5 * k_2, y_0 + 0.5 * p_2, z_p, z_t)$
16. $p_3 = h * fx1(v_{x0} + 0.5 * k_2)$
17. $k_4 = h * fx(t_0 + h, x_0 + k_3, y_0 + p_3, z_p, z_t)$
18. $p_4 = h * fx1(v_{x0} + k_3)$
19. $v_{x1} = v_{x0} + (k_1 + 2 * k_2 + 2 * k_3 + k_4)/6.0$
20. $x_1 = x_0 + (p_1 + 2 * p_2 + 2 * p_3 + p_4)/6.0$
21. $k_1 = h * fy(t_0, x_0, y_0, z_p, z_t)$
22. $p_1 = h * fy1(v_{y0})$
23. $k_2 = h * fy(t_0 + 0.5 * h, x_0 + 0.5 * k_1, y_0 + 0.5 * p_1, z_p, z_t)$
24. $p_2 = h * fy1(v_{y0} + 0.5 * k_1)$

25. $k_3 = h * fy(t_0 + 0.5 * h, x_0 + 0.5 * k_2, y_0 + 0.5 * p_2, z_p, z_t)$

26. $p_3 = h * fy1(v_{y0} + 0.5 * k_2)$

27. $k_4 = h * fy(t_0 + h, x_0 + k_3, y_0 + p_3, z_p, z_t)$

28. $p_4 = h * fy1(v_{y0} + k_3)$

29. $v_{y1} = v_{y0} + (k_1 + 2 * k_2 + 2 * k_3 + k_4)/6.0$

30. $y_1 = y_0 + (p_1 + 2 * p_2 + 2 * p_3 + p_4)/6.0$

31. $dist = sqrt(x_1 * x_1 + y_1 * y_1)$

32. $dist = dist * 10e - 15$

33. $if(dist < dmin) dmin = dist$

34. Print t,$(x_1 + x_t), and\ (y_1 + y_t)$

35. $vx_0 = v_{x1}$

36. $x_0 = x_1$

37. $v_{y0} = v_{y1}$

38. $y_0 = y_1$

39. $t_0 = t$

40. End for t

41. Print the distance of closest approach, *dmin.*

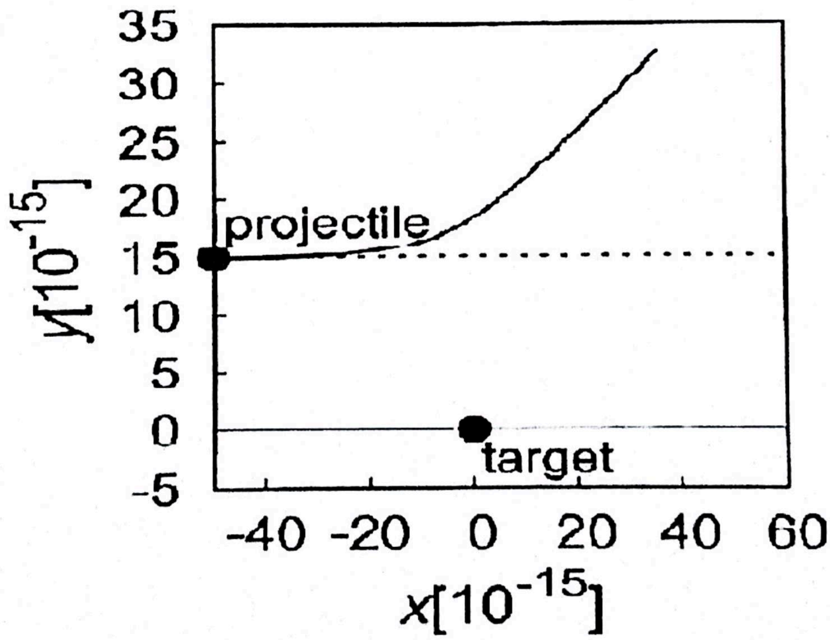
42. stop

PROGRAM:

```
//Rutherford scattering of alpha partcile
#include<stdio.h>
#include<math.h>
double fx(double t,double x,double y,double zp,double zt);
double fy(double t,double x,double y,double zp,double zt);
#define fx1(vx) (vx)
#define fy1(vy) (vy)
#define B (0.3471208)
main()
{
double k1,k2,k3,k4,p1,p2,p3,p4, x1,y1,x0,y0,h=0.01,tmax=25.0,t0,t;
double zp=2,zt=79;//atomic numbers of alpha particle and gold
double vx0,vx1,vy0=0,vy1,E,ma,dmin=50,dist;
double xt=0,yt=0,xp=-50,yp=15; //position of target & projectile
FILE *fp;
fp=fopen("rutherford.dat","w");
printf("Enter energy of alpha-particle in ev\n");
scanf("%le",&E);
E=E*1.6*10e-19;//convert energy in joules
ma=6.645e-27;// mass of alpha-particle
vx0=sqrt(2*E/ma);//calculate velocity of alpha-particle
vx0=vx0*10e-7;//scale velocity
x0=xp-xt;//relative vector
y0=yp-yt;
dist=sqrt(x0*x0+y0*y0);//distance of alpha-particle from projectile
dist=dist*10e-15;
fprintf(fp,"%lf\t%lf\t%lf\n",t0,xp,yp);
for(t=t0+h;t<=tmax;t+=h)
{
k1=h*fx(t0,x0,y0,zp,zt);
k2=h*fx(t0+0.5*h,x0+0.5*k1,y0+0.5*p1,zp,zt);
p2=h*fx1(vx0+0.5*k1);
k3=h*fx(t0+0.5*h,x0+0.5*k2,y0+0.5*p2,zp,zt);
p3=h*fx1(vx0+0.5*k2);
k4=h*fx(t0+h,x0+k3,y0+p3,zp,zt);
p4=h*fx1(vx0+k3);
vx1=vx0+(k1+2*k2+2*k3+k4)/6.0;
x1=x0+(p1+2*p2+2*p3+p4)/6.0;
k1=h*fy(t0,x0,y0,zp,zt);
p1=h*fy1(vy0);
k2=h*fy(t0+0.5*h,x0+0.5*k1,y0+0.5*p1,zp,zt);
p2=h*fy1(vy0+0.5*k1);
k3=h*fy(t0+0.5*h,x0+0.5*k2,y0+0.5*p2,zp,zt);
p3=h*fy1(vy0+0.5*k2);
k4=h*fy(t0+h,x0+k3,y0+p3,zp,zt);
p4=h*fy1(vy0+k3);
vy1=vy0+(k1+2*k2+2*k3+k4)/6.0;
y1=y0+(p1+2*p2+2*p3+p4)/6.0;
dist=sqrt(x1*x1+y1*y1);
dist=dist*10e-15;
if(dist<dmin)dmin=dist;
```

```
fprintf(fp,"%lf\t%lf\t%lf\n",t,(x1+xt),(y1+yt));
vx0=vx1;
x0=x1;
vy0=vy1;
y0=y1;
t0=t;
}
printf("distance of closest approach=%le meter\n",dmin);
fclose(fp);
}
double fx(double t,double x,double y,double zp,double zt)
{ return(B*zp*zt*x/sqrt((x*x+y*y)*(x*x+y*y)*(x*x+y*y)));
}
double fy(double t,double x,double y,double zp,double zt)
{ return(B*zp*zt*y/sqrt((x*x+y*y)*(x*x+y*y)*(x*x+y*y)));
}
```

```
/**************OUTPUT************************
Enter energy of alpha-particle in ev
40000
distance of closest approach=1.775700e-13 meter
************************************************/
```



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Thank you